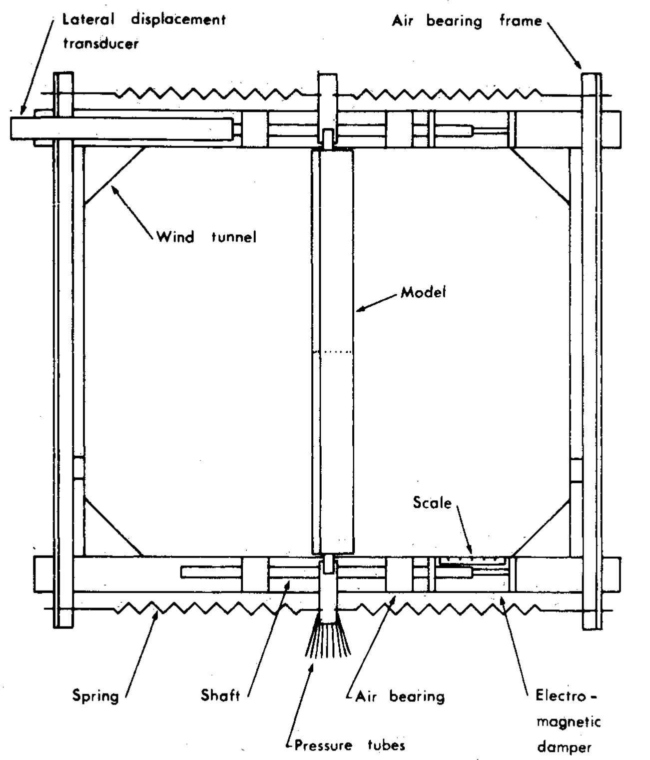

Fig. 2 Details of model support system permitting plunging degree of freedom.

For higher wind speeds, say $U^* > 4$, the frequency of vortex shedding has becomes sufficiently well separated from the cylinder frequency so as to have no influence on the galloping instability.

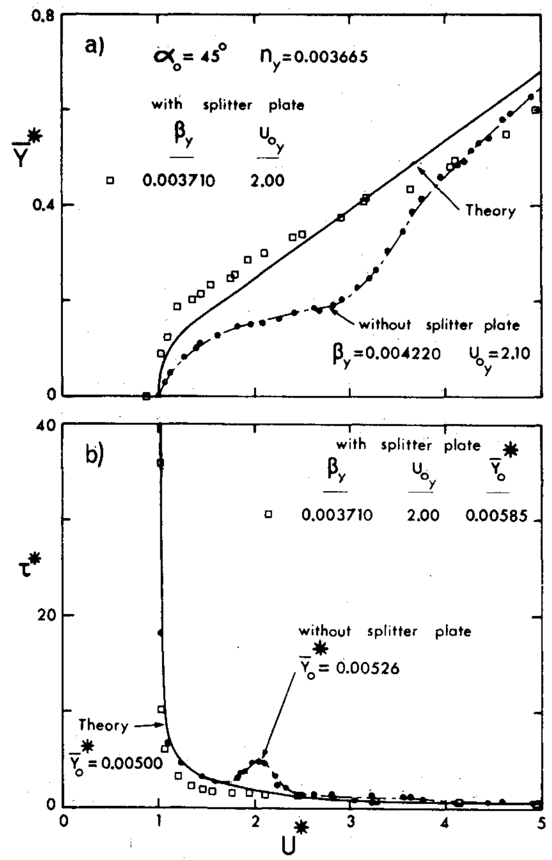

Fig. 3 Dimensionless plots of galloping amplitude and build-up time vs velocity showing the effect of vortex suppression through the use of a splitter plate.

The lower amplitude motion exhibited by the angle section for $U^* < 4$ in absence of the splitter plate may represent partial asynchronous quenching[5] of the galloping oscillations by the vortex induced fluctuating forces. Parkinson and Santosham[6] have extended the quasi-steady galloping theory to include the effects of the wake vortices and predicted asynchronous quenching for a square section in water.

It should be mentioned that the change in slope of the asymptote for the oscillating angle section with the splitter plate is not too significant because only small amplitude measurements can be obtained with the plate held stationary. Large relative motions between model and plate would create a different flowfield and consequently a variation in the resulting aerodynamic forces.

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Shock Wave Shaping

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Introduction

RECENT work[1,2] has brought attention to the shaping of strong shock waves by suitable wall shaping. The motivation for this is primarily the use of area changes in a duct to produce stronger shocks, and hence high enthalpy gas which may then be expanded in the nozzle of a short duration hypersonic tunnel. The above two references describe attempts to generate a shaped collapsing wave front to achieve this purpose. Both methods use Whitham's approximate theory[3] for strong shocks to establish the wall profile. In the first case the characteristic wave is centred and in the second the wall shape is arranged to result in a cylindrical wave. In neither case is a comparison made between the experimental and theoretical shock profiles. This Note describes some work which uses similar methods but, since weaker shocks are involved, employs the more general solution to Whitham's theory.

Theoretical Considerations

Whitham's theory for determining shock profiles employs the method of characteristics and ignores any effect of the flowfield behind the shock on the shock itself. It assumes that the shock propagates along ray-tubes within which there is a differential relationship between the shock Mach number and ray-tube area.

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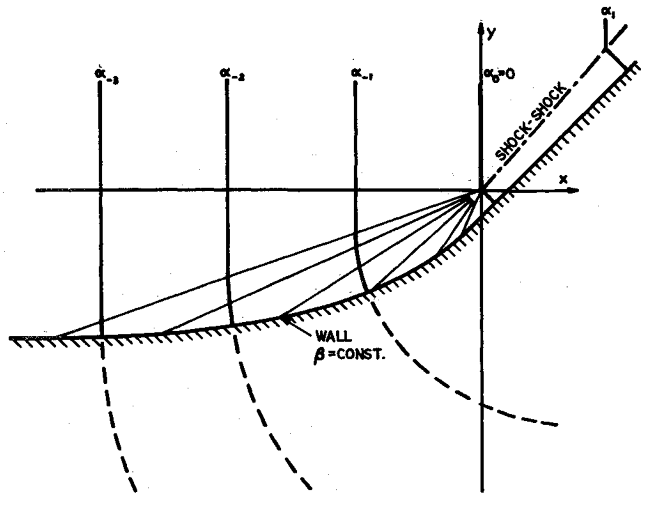

Fig. 1 Characteristic net for a centered simple wave.

The shock Mach number is constant along each of the characteristics. The original paper should be consulted for details.

The characteristic net for a centred simple wave, which is the case being considered, is shown in Fig. 1. The wall is a ray denoted by a constant value of β and the shock positions are curves of constant α where $\alpha = a_o t$, and a_o is the sound speed in the undisturbed region ahead of the shock and t is the time taken by the shock to move from any given position to a position which passes through the origin. The curvature of the shock increases as the shock progresses until at the singularity, where the characteristics intersect, it becomes infinite. Thus a discontinuity in slope and Mach number of the shock occurs at this point and the subsequent motion represents that of Mach reflection.

It has been shown previously[4] that the shape of the shock within the simple wave is given by

$$x/\alpha = M\cos(\theta + m)/\cos m, \quad y/\alpha = M\sin(\theta + m)/\cos m \qquad x, y, \alpha < 0$$

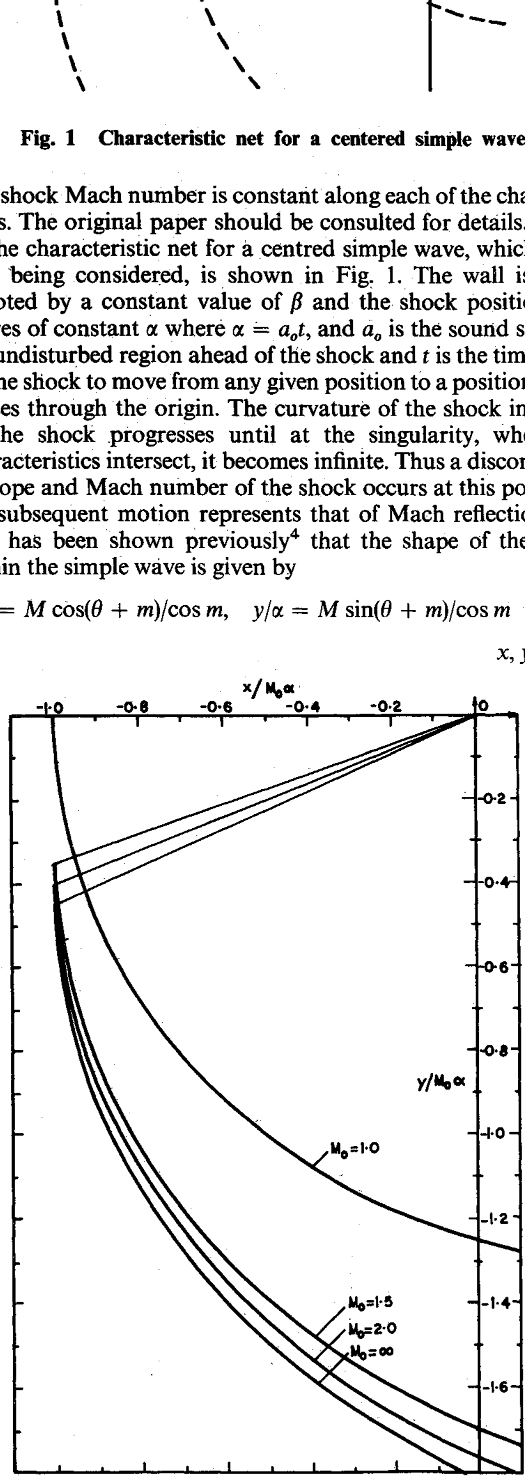

Fig. 2 Results for incident shock Mach numbers of 1.0, 1.5, 2.0, and ∞.

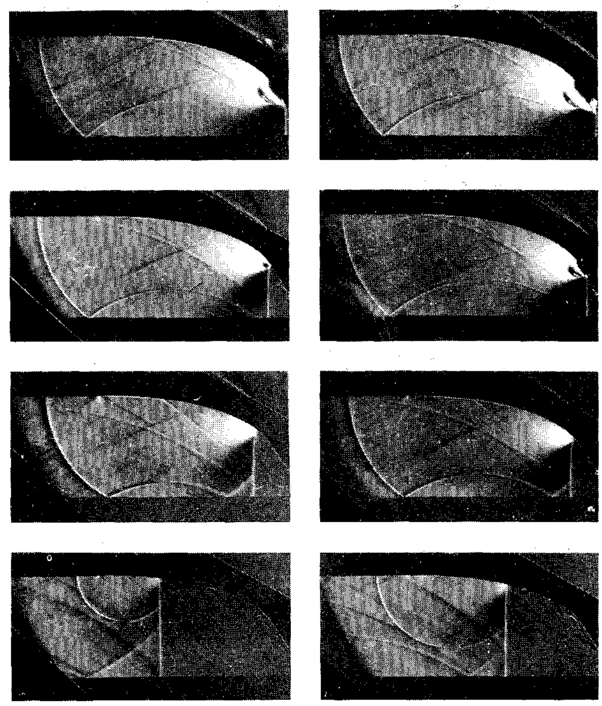

Fig. 3 Schlieren results.

M is the Mach number along the characteristic; θ is the inclination of a ray to the x axis and depends on the change in Mach number along a ray through an equation analagous to the Prandtl-Meyer relation in steady flow; and m is the angle between a ray and a characteristic and is also a function of M.

The equation of the wall, β = constant < 0, is given by

$$x/\beta = A\cos(\theta + m)/\sin m, \quad y/\beta = A\sin(\theta + m)/\sin m$$

where A is the ray-tube area. Curves of constant α and constant β are orthogonal. For the strong shock case the angle m between the wall, or any ray, and the characteristics is assumed constant, and the wall, and the shock, are thus logarithmic spirals.

The shock profiles within the simple wave are self-similar in time and thus a single curve may be drawn for a given shock strength and for all times by dividing the shock coordinates by α. Results for incident shock Mach numbers of 1.0, 1.5, 2.0, and ∞ are given in Fig. 2. For convenience the axes are $x/M_o\alpha$, $y/M_o\alpha$ in order to place the incident shock in the same position for all cases.

Experimental Results

An accurately shaped wall was manufactured and tested in a 3 in. × 2 in. shock tube. The wall profile chosen was for $M_o = 1.5$, total turning angle 57.3° and $\beta = -20$ in. These initial conditions theoretically result in a wall shock Mach number immediately after the bend of $M_w = 2.2$. The schlieren results are given in Fig. 3. Superimposed tracings from these records and the theoretical predictions are given in Fig. 4. For convenience the origin in this latter figure is taken at the start of the wall curvature. This new origin is situated at a point with coordinates (-3.38 in., -1.18 in.) from the previous origin.

A slight step was arranged at the point where the curvature of the wall starts so that the sound wave generated at the step, on passage of the shock, would clearly show on the records and thus would indicate which part of the flowfield had been influenced by signals from the wall.

Discussion

The locus of the intersection of the first reflected sound wave and the incident shock is the first characteristic of the fan which carries changes to Mach number to the shock. The difference between this characteristic and the equivalent characteristic in Whitham's theory has been dealt with elsewhere.[5]

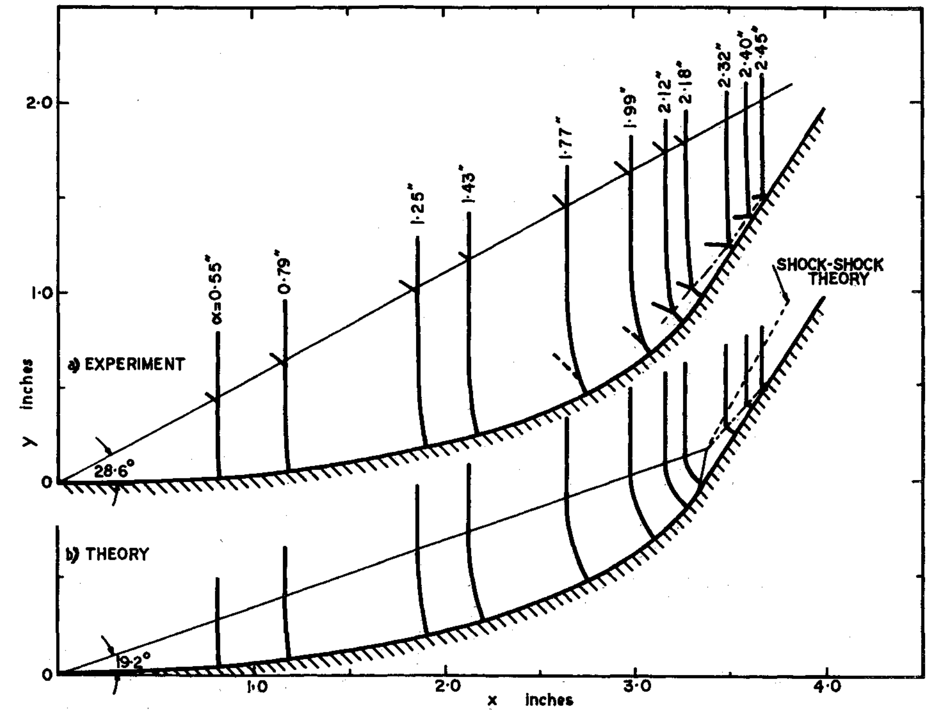

Fig. 4 Theoretical predictions.

Even in the early stages of the process the photographs show a high density gradient immediately behind the shock in proximity to the wall. This compression wave becomes better defined as the shock progresses until at about $\alpha = 2.15$ in. it becomes a reflected shock and a Mach reflection results. This Mach reflection is of the inverted type in that the triple point moves towards the wall and the reflection eventually becomes regular.

The shock-shock shown in Fig. 4b was obtained from geometrical considerations by knowing that the wall shock has a Mach number of 2.2 as predicted from the simple wave theory. On the other hand Whitham's shock-shock theory[3] does not permit the shock-shock to approach the wall as this would permit rays that come from each side of the start of the shock-shock, to intersect. This is inadmissable in the theory. There is thus an anomaly in the end conditions given by the simple wave theory and the conditions required by the shock-shock theory.

The surprising result shown by these tests is that notwithstanding the difference in the starting point of shock curvature nor the noncentred nature of the wave, Whitham's theory gives good agreement as to the position where the shock-shock starts, and as to the position of the shock-shock established on the basis of simple geometrical considerations.

A further interesting point to notice is that the shock curvature is mainly confined to the lower portion of the shock; to such an extent that at $\alpha = 2.45$ in. the shock cannot be distinguished from being plane. Changes in Mach number carried by the upper characteristics of the fan are thus evidently small.

A comparison between the theoretical and experimental shock profiles shows similar effects to previous comparisons for a shock diffracting on plane walls.[5] The experimentally obtained curvature is less than the theoretical so that the part of the shock in contact with the wall is weaker than expected from the theory. These previous tests also showed that the agreement is better for stronger shocks ($M_o > 3.0$) and the same would be expected for shocks shaped in the manner described here.

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Numerical Calculation of Sharp Flat Plate Transitional and Turbulent Skin Friction

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Nomenclature

C_{f_e} = local skin-friction coefficient based on boundary-layer edge conditions, $2\tau_w/\rho_e U_e^2$
M_e = local edge Mach number
$Re_{e,x}$ = local Reynolds number based on boundary-layer edge conditions, $\rho_e U_e x/\mu_e$
T_{aw} = adiabatic wall temperature
T_w = wall temperature
U_e = streamwise velocity at edge of boundary layer
x = surface distance measured from apex of plate
μ_e = viscosity evaluated at boundary-layer edge temperature
ρ_e = density evaluated at boundary-layer edge pressure and temperature
τ_w = wall shearing stress

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